Disordered one-dimensional contact process

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(Received 10 December 1997)

Theoretical and numerical analyses of the one-dimensional contact process with quenched disorder are presented. We derive scaling relations which differ from their counterparts in the pure model, and that are valid not only at the critical point but also away from it due to the presence of generic scale invariance. All the proposed scaling laws are verified in numerical simulations. In addition, we map the disordered contact process into a non-Markovian contact process by using the so called run time statistics, and write down the associated field theory. This turns out belong to the same universality class as the one derived by Janssen [Phys. Rev. E **55**, 6253 (1997)] for the quenched system with a Gaussian distribution of impurities. Our findings reported herein support the lack of universality suggested by the field-theoretical analysis: generic power-law behaviors are obtained. We moreover show the absence of a characteristic time away from the critical point, and the absence of universality is put forward. The intermediate sublinear regime predicted by Bramsom, Durret, and Schnmann [Ann. Prob. **19**, 960 (1991)] is also found. [S1063-651X(98)04905-8]

PACS number(s): 05.50.+q, 02.50.−r

I. INTRODUCTION

As first conjectured by Janssen and Grassberger [1], many numerical and analytical studies have established clearly that all systems exhibiting a continuous transition into a *unique* absorbing state, without any other extra symmetry or conservation law, belong to the same universality class, namely, that of the contact process [2,3]. This conjecture has been extended to include multicomponent systems [4], and also systems with an infinite number of absorbing states [5]. Among many other models in this broad class are directed percolation [2,3], the contact process [6], catalytic reactions on surfaces [7], the spreading of epidemics, and branching annihilating random walks [8]. Reggeon field theory is the minimal continuous theory capturing the key features of this universality class [9,1] [which is often referred to as the directed percolation (DP hereafter) universality class].

Despite its theoretical importance, no experiment has succeeded so far in identifying critical exponents compatible with the predicted DP values. This could be due to the fact that real systems are never pure, i.e., they present impurities, dilution, or other forms of disorder. The question arises of how disorder affects the critical behavior of DP-like systems. That problem was first posed by Kinzel [10] and studied numerically by Noest [11,12], who showed using a Harris criterion [13], that quenched disorder changes the critical behavior of DP systems in spatial dimensions below $d=4$. He also demonstrated that when $d=1$, a generic power law (generic scale invariance) can be observed, and that in $d=2$ a Griffiths-like phase [14] can appear when the impurities take the form of dilution [11]. This same problem was recently tackled by Dickman and Moreira in interesting papers [15,16], where they pointed out the presence of logarithmic time dependences in the $d=2$ case, and a possible violation of scaling.

Two other related problems are temporarily disordered systems with absorbing states, which have also been recently investigated, with apparently striking conclusions [17]; and the problem of heterogeneous catalysis on disordered media, for which a much richer behavior is observed than in its counterpart without disorder [18].

In any case, the dynamics in impure DP systems is well established to be extremely slow. Due to the presence of impurities, a system that is globally in the absorbing phase can include regions that locally take parameter values that correspond to the active regime in the analogous pure system. The presence of these regions prevents the system from easily relaxing to the absorbing state, and consequently it decays in a slow fashion, i.e., exhibiting power laws in $d=1$ [11,12], and logarithmically in $d=2$ [15,16], but not exponentially, as generically expected in pure systems away from the critical point.

At a theoretical level a field-theory analysis for this class of impure systems was recently derived by Janssen [19]. This work corrects a previous incomplete analysis [20], and concludes from an ϵ expansion around the upper critical dimension, $d=4$, that the renormalization group flow equations exhibit only runaway trajectories, and that therefore there is no stable (perturbative) fixed point (nothing can be concluded about nonperturbative fixed points). This can be seen as evidence that no universal critical behavior is expected in this class of models.

In this paper we revisit the impure one-dimensional problem, and look at it within an interesting perspective. In particular, we analyze the presence or absence of scaling laws in analogy to the two-dimensional results recently presented by Dickman and Moreira. We moreover study the universality of critical exponents and the scaling relations they obey, and verify the presence of a sublinear regime predicted by Bramson, Durret, and Schnmann [21]. On the other hand, we present a non-Markovian representation of this class of systems that shows the same phenomenology. This approach enables us to derive a field theory that turns out to be equivalent to the one derived by Janssen. From the field theory, we finally obtain relations among exponents.

II. MODEL

In the standard contact process [6,2] each site of a d-dimensional lattice is either ''occupied'' or ''vacant.'' In its discrete-time version, an occupied site is extracted randomly at each time step; it generates an offspring with probability p, or disappears with complementary probability $1-p$. The offspring occupies a randomly chosen nearest neighbor; if it was empty it becomes occupied, while the system remains unchanged if the neighbor was already occupied. In the disordered contact process the probability p changes from site to site. It is fixed in time, and obeys to a distribution $\Pi(p)$. Through this paper we consider in particular

$$\Pi(p,a)=ap^{a-1}, \tag{1}$$

for which

$$\langle p\rangle=\frac{a}{a+1}; \tag{2}$$

in this way a acts as a control parameter. For large values of a, the creation probability is large, and the system is in the active phase, while for sufficiently small values of p the system decays into the absorbing state. We have chosen the previous distribution for a technical reasons: it simplifies the application of the run time statistic [22–24] that we use to study the model.

The central magnitudes usually considered in this kind of system are of two types: magnitudes measured in analysis with homogeneous initial conditions, and those measured studying the spreading of a localized ''seed'' into the otherwise empty space [25]. In the first group, we determine the stationary order parameter n (defined as the average density of particles in the stationary state), the correlation time τ, and the correlation length ξ. In the second group we study (i) the total number of occupied sites in the lattice (averaged over all the runs including those which have reached the absorbing state) as a function of time, $N(t)$; (ii) the overall surviving probability $P_s(t)$, corresponding to the probability that the system has not reached the absorbing state at time t, and (iii) the mean square distance of spreading from the origin for the trials still surviving at at a given time t as a function of time, $R^2(t)$.

Right at the critical point of pure systems, we have

$$N(t)\propto t^{\eta}, \quad P_s(t)\propto t^{-\delta}, \quad R^2(t)\propto t^{z} \quad \text{and} \quad n(t)\propto t^{-\theta}, \tag{3}$$

and, at a small distance Δ from the critical point,

$$n\propto\Delta^{\beta}, \quad \tau\propto\Delta^{-\nu_t}, \quad \xi\propto\Delta^{-\nu_x}, \tag{4}$$

which define the set of critical exponents we are interested in. In pure systems the following scaling relations hold:

$$\eta+\delta+\theta=dz/2, \quad \delta=\theta, \quad z=2\nu_x/\nu_t \quad \text{and} \quad \theta=\beta/\nu_t; \tag{5}$$

these expressions have to be modified for the disordered model as we will show (see Refs. [3,1,26], and references therein).

III. NON-MARKOVIAN REPRESENTATION

We start our analysis of the model by mapping it into a non-Markovian model. The idea of representing a model with quenched disorder by means of an effective non-Markovian equation, i.e., with memory, including no disorder, is not totally new. A complete theory that justifies such an approach was developed in Ref. [22]; it was named the run time statistic (RTS), and has proven to be a useful tool in both the study of fractals with quenched disorder [23] and self-organized models with extremal dynamics [24].

The central idea of the RTS can be exemplified by its application to the random random walker (RRW) [27]. The RRW is defined in the following way: a standard one-dimensional random walker is considered, with the only difference being that the probabilities of jumping to the right, q, or to the left, $1-q$, change from site to site, are quenched, and extracted from a certain probability distribution $P(q)$. The probability that at a given site, characterized by a given value of q, visited n times by the walker, the walker has jumped k times to the left, is easily shown to be given by the binomial distribution [27]

$$P(k|q,n)=\frac{n!}{k!(n-k)!}q^k(1-q)^{n-k}. \tag{6}$$

Using the Bayes inversion formula for the inversion of conditional probabilities, one can calculate the probability that at a given site the probability q takes a particular value between q and $q+dq$ from the knowledge of k after n jumps [28,27]:

$$P(q+dq|n,k)=\frac{(n+1)!}{k!(n-k)!}q^k(1-q)^{n-k}P(q)dq. \tag{7}$$

An effective transition probability can be accordingly defined as

$$q(n,k)=\int dq\ qP(q|n,k). \tag{8}$$

This equation gives the effective probability for the walker to jump to the right in its $n+1$ visit to a given site, conditioned to the fact that in n previous visits it jumped k times to the right.

Observe that the distribution equation (7) changes with time (i.e., with n); the information about the history of the system is contained in the effective transition probabilities (that change from site to site). This is usually called *run time statistics* [22].

Let us now apply the previously described method to the disordered contact process. At each site the value of p (that plays now a role analogous to q in the RRW), is extracted from the distribution equation (1) [29]; it is straightforward to verify that

$$P(p+dp|n,k)=p^{k+a-1}(1-p)^{n-k}\frac{(n+1)!}{(k+a-1)!(n-k)!}dp, \tag{9}$$

where n is the number of times that a given site has been chosen to try an evolution step, and k is the total number of times in which an offspring has been generated (obviously $n-k$ is the number of events in which the site has become empty). Therefore the effective parameter p at the site under consideration is

$$\langle p\rangle=\int_0^1 dp\ pP(p+dp|n,k)=\frac{k+a}{n+a+1}. \qquad (10)$$

Note that in the infinite n limit the distribution of effective values of the probability $\langle p\rangle$ does not collapse to a δ function, but converges asymptotically to the distribution equation (1) [27].

IV. FIELD THEORY

Using the previously derived non-Markovian approach, we can construct an associate field theory. Let us first consider the standard Reggeon field theory describing the universality class of the pure contact process [1,9]:

$$S[\phi,\psi]=\int dr^d\int_0^\infty dt\{\lambda\psi(x,t)^2\phi(x,t)-\psi(x,t)[\partial_t\phi-\mu^2\phi -\lambda\phi(x,t)^2-\nabla^2\phi(x,t)]\}. \qquad (11)$$

The coefficient of the linear term, μ^2 (the *mass* in a field theoretical language), depends linearly on the creation probability p. A large p makes the contact process supercritical. So does a value of μ^2 above its critical value. Therefore in order to implement the idea of a $\langle p\rangle$ value changing in time [as happens in Eqs. (9) and (10)] in the field theory, we should consider a time dependent μ^2. To see how μ^2 should be modified, one should observe that at any time the renormalized value of μ^2 at a given point x is given by the expectation value of $\psi(x,t)\phi(x,t)$. Therefore, in order to introduce the dependence of p on the system history at each point x, we can perform the following substitution:

$$\mu^2\to\mu^2_{\rm mod}(x,t)=\mu^2+\gamma\int_0^t d\tau\ \psi(x,\tau)\phi(x,\tau); \qquad (12)$$

that is, at every time step, the *modified* value of the linear coefficient, $\mu^2_{\rm mod}$, is given by its original value corrected by a time dependent term given by the expectation value of $\psi(x,t)\phi(x,t)$ integrated over the previous history of the system, i.e., the original creation probability is substituted by a sort of time average of the creation probability along the previous system path (observe that γ acts as a normalization factor), in analogy to what we obtained in Eqs. (9) and (10). We want to stress that the construction here is not rigorous but only a reasonable guess based on the knowledge derived from the run time statistics approach.

By performing the previous substitution, the action becomes

$$S_M[\phi,\psi]=\int dr^d\int_0^\infty dt\Big[\lambda\psi^2\phi-\psi(\partial_t\phi-\mu^2\phi-\lambda\phi^2 -\nabla^2\phi)+\gamma\psi\phi\int_0^t d\tau\ \psi(x,\tau)\phi(x,\tau)\Big], \qquad (13)$$

where the dependence of the fields on x and t has been omitted to simplify notations.

On the other hand, considering the standard Reggeon field theory [Eq. (11)], with a site dependent quenched mass coefficient $\mu^2(x)$, and a Gaussian distribution of "masses" with mean μ^2, and variance $\langle\mu^2(x)\mu^2(x')\rangle=(f/2)\delta(x-x')$, after averaging over the disorder (one just has to perform a Gaussian integral) [19] one obtains

$$S_d[\phi,\psi]=\int dr^d\Big[\int_0^\infty dt[\lambda\psi^2\phi-\psi(\partial_t\phi-\mu^2\phi-\lambda\phi^2 -\nabla^2\phi)]\Big]+f\Big[\int_0^\infty dt\ \psi(x,t)\phi(x,t)\Big]^2. \qquad (14)$$

Observe that Eq. (14) is very similar to Eq. (13), except for the time integration limit and the value of the multiplicative coefficients of the second term on the right-hand side. This difference states that only in the large time limit does the non-Markovian approach reproduce the exact result (as also happens in the run time statistics approach). Therefore, we take the infinite time limit in the upper limit of the integral equation (12), and in this way we recover Eq. (13).

Naive power counting arguments show that all three nonlinearities in Eq. (13) can be renormalized in $d=4$. This result is consistent with the Harris criterium [13] presented by Kinzel [10] and Noest [11], which states that quenched spatial disorder affects the critical behavior of the contact process and models in the same universality class below $d=4$. The detailed renormalization procedure of Eq. (14) can be found in Ref. [19].

V. SCALING LAWS

The field theory we have written down can also be used as a starting point to derive scaling relations. From Eq. (11) (or using other standard scaling arguments), it is easy to derive that, in the active regime,

$$\eta+\delta+\theta=dz/2. \qquad (15)$$

Let us derive the corresponding relation for the impure (non-Markovian) model here. We invoke the simple arguments that, as $N(t)$ is obtained averaging over all the runs, it can be written as $N(t)=N_s(t)P_s(t)+0\times[1-P_s(t)]$ where $N_s(t)$ is the total number of particles calculated averaging only over surviving runs. Consequently, one obtains $N_s(t)\approx t^{\eta+\delta}$. After creating a perturbation, if a growing cluster of occupied sites is generated, the radius of such a cluster grows as $R\propto t^{z/2}$, and its volume as $R^d\propto t^{dz/2}$. From the two previous expressions, the density of particles inside the cluster goes like $t^{\eta+\delta-dz/2}$. But the density inside the cluster scales as $t^{-\theta}$ by definition of θ. Therefore, we straightforwardly obtain Eq. (15).

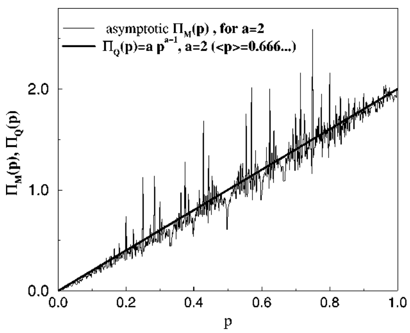

FIG. 1. Distribution of $\langle p\rangle$ for large times in the non-Markovian model $\Pi_M(p)$ compared with the fixed distribution of the disordered model $\Pi_Q(p)$ for $a=2$.

The previous expression is valid only at the critical point in the pure model, where scale invariance is expected. Contrarily, in the impure model, where generic scale invariance is expected, the previous argument is valid in all the active phase, in which growing clusters are typically generated from localized seeds.

On the other hand, in the absorbing phase, typically initial seeds are located in locally absorbing regions and die out exponentially. However, there is a probability for the initial seed of ''landing'' in a locally active cluster. When the perturbation escapes from these clusters, it dies out exponentially more quickly. But, inside these finite clusters, the local stationary density is reached in a finite time. Therefore we can substitute formally θ by zero in Eq. (15), and obtain

$$\eta+\delta=dz/2 \tag{16}$$

(note that this does not mean that θ is zero).

On the other hand, using the symmetry of the Lagrangian under the exchange of the fields ϕ and ψ, it is not difficult to obtain

$$\delta=\theta, \tag{17}$$

as in the pure model (see Ref. [26] for a review of the underlying ideas). Observe that the previous symmetry, present in the Reggeon field theory, is not broken by the introduction of the non-Markovian term, i.e., by the quenched impurities. Therefore,

$$\eta+2\delta=dz/2 \tag{18}$$

in the active regime of our model, as well as in the critical point of the pure model.

In the active phase, starting from an homogeneous distribution, the system also relaxes to its stationary state as a power law. By definition of the active regime, the surviving probability does not go to zero for large times. Moreover, as $P_s(t)$ is a monotonously decreasing function of time, we obtain that $\delta=0$ all along the active phase. Using the scaling relation presented in Eq. (17), we also obtain $\theta=0$. This simplifies Eq. (18) to

$$\eta=dz/2. \tag{19}$$

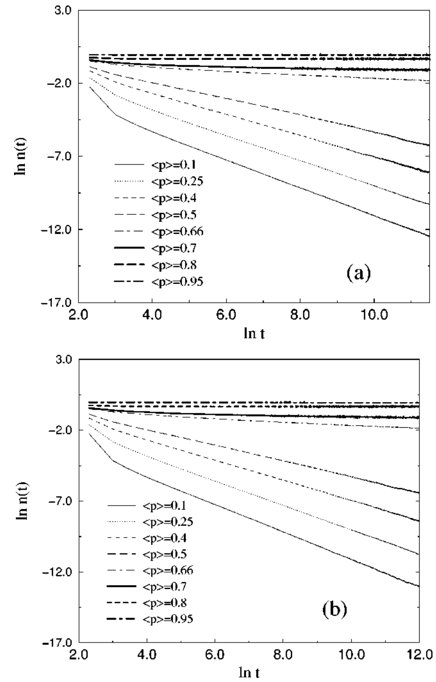

FIG. 2. Decay of the density of occupied sites as a function of time for the disordered model (upper plot) and for the non-Markovian model (lower plot), for different values of $\langle p\rangle$.

For completeness, let us point out that the exponent $\hat{d}$ calculated in Ref. [11] is easily related to the exponents we have defined, since

$$\hat{d}=1+\eta+\delta. \tag{20}$$

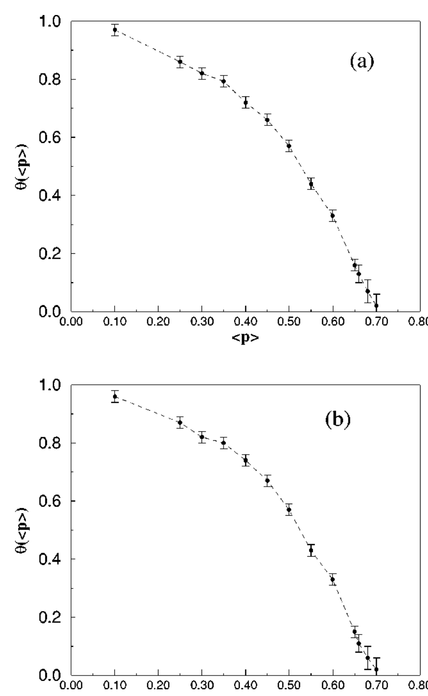

FIG. 3. Value of the exponent $\theta(\langle p\rangle)$ as a function of $\langle p\rangle$ for the disordered model (upper plot) and for the non-Markovian model (lower plot).

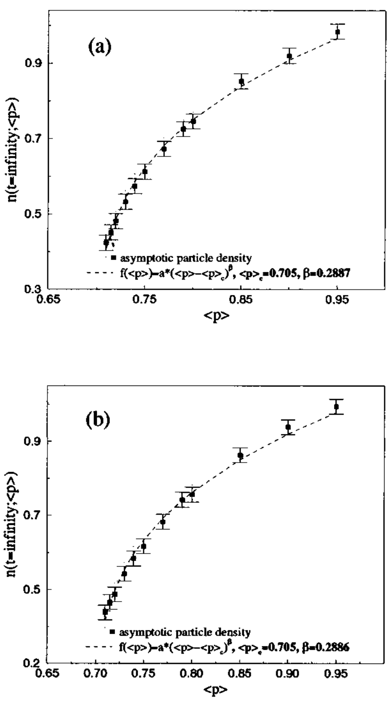

FIG. 4. Stationary value of the density as a function of $\langle p\rangle$ for the disordered model (upper plot) and for the non-Markovian model (lower plot). A power-law fit for the scaling of $n(t=\infty;\langle p\rangle)$ is shown in the figure.

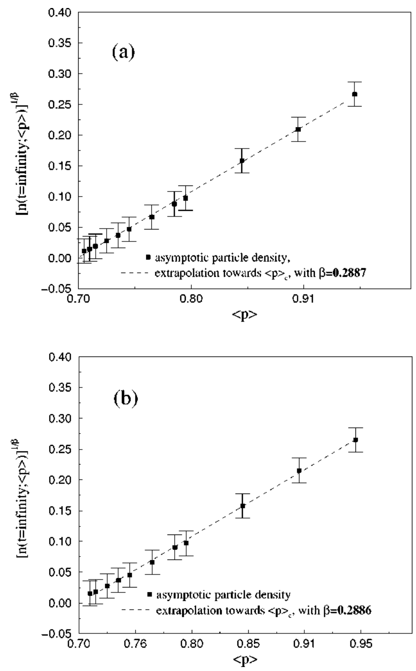

FIG. 5. Log-log plot of $n(t=\infty;\langle p\rangle)^{1/\beta}$ as a function of $\langle p\rangle$ for the disordered model (upper plot) and for the non-Markovian model (lower plot). The value of β is that given by the fit in Fig. 4. The extrapolation to $n(t=\infty)=0$ gives $p_c=0.71\pm0.01$.

To justify this, it is sufficent to observe that $\hat{d}$ is the exponent of a time integral of the total number of particles averaged over the surviving runs.

Summing up, the main conclusions of this section are the following: (i) In the active phase, $\eta=dz/2$ and $\delta=\theta=0$. (ii) In the absorbing phase, $\eta+\delta=dz/2$ and $\delta=\theta\neq0$.

VI. MONTE CARLO RESULTS

We have performed extensive Monte Carlo simulations of the contact process with quenched impurities distributed according to Eq. (1), as well as of the associated non-Markovian contact process defined by Eqs. (9) and (10). Spreading experiments have been performed in lattices that were large enough to ensure that the occupied region does not reach the system limits. Experiments are started with random homogeneous initial conditions and are performed in system sizes up to $L=10^4$ and periodic boundary conditions. But most of the results presented correspond to $L=10^3$. At every time step a particle is randomly chosen, and the dynamics proceeds in the way explained in Sec. II; after each step the time variable t is increased in $1/N(t)$; i.e., when all the particles are updated once on average, the time increases in one unit. Simulations are run long enough to ensure that the system relaxes to its stationary state in the active phase ($t\approx1.6\times10^5$ time steps). The different magnitudes are obtained by averaging over many independent runs (from 10^2 for large values of $\langle p\rangle$, where most realizations die at late times and we can easily collect a good statistics, to 10^5 for small values of $\langle p\rangle$, where many realizations die at early times). All the forthcoming discussions are valid for both the model with quenched disorder and the non-Markovian equivalent model; the results coincide within the numerical accuracy. In fact, as it is shown in Fig. 1, the long time distribution of values of $\langle p\rangle$ in the non-Markovian model is verified to converge to the distribution in the quenched model of Eq. (1). To avoid repetition, we discuss both cases as a whole, and present figures for both the disordered and non-Markovian models. We now present the main results we have obtained.

A. Homogeneous initial conditions

The density of particles $n(t)$ $[n(t)=N(t)/L]$ decays in time as shown in Fig. 2. Observe that for large enough values of $\langle p\rangle$ the curves converge to a stationary value, that is, their derivative with respect to time converges to zero asymptotically. On the other hand, for small values of $\langle p\rangle$ the curves decay like power laws with nonuniversal exponents that depend on $\langle p\rangle$; $n(t)\propto t^{-\theta(<p>)}$. In Fig. 3, we show the asymptotic exponent $\theta(\langle p\rangle)$ as a function of $\langle p\rangle$, for both the disordered and non-Markovian models. It decays continuously from its maximum value in the absorbing state to a very small value (compatible with zero). It vanishes in the active regime. Also observe the difficulty in accurately locating the critical point. Usually, in pure systems away from the critical point, $n(t)$ decays exponentially in the absorbing phase, and converges to a constant value in the active phase. It decays as a power law only at the critical point. Consequently there is a neat criterion to identify the critical point: power laws are the signature of criticality. In the impure model, instead, the generic presence of power laws makes the determination of the critical point a more delicate issue, but at the same time a more irrelevant one.

Two possible scenarios are compatible with the data we have obtained: in the first one $\theta(\langle p\rangle)$ is a continuous func-

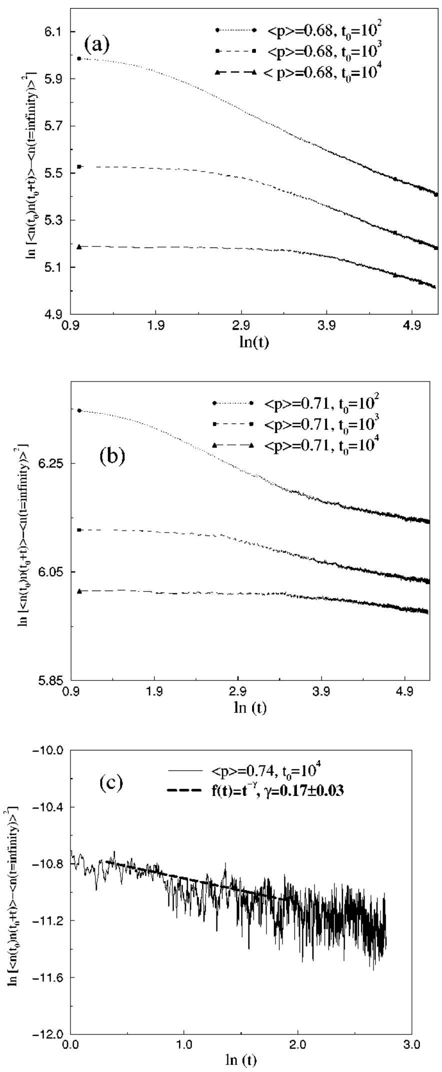

FIG. 6. Decay of the two-time density correlation function for different initial times (disordered model) as a function of the time difference t, and different values of $\langle p\rangle$: $\langle p\rangle=0.68$ in the absorbing phase (upper figure), $\langle p\rangle=0.71$ at critical point (central figure), and $\langle p\rangle=0.74$ in the active phase (lower figure).

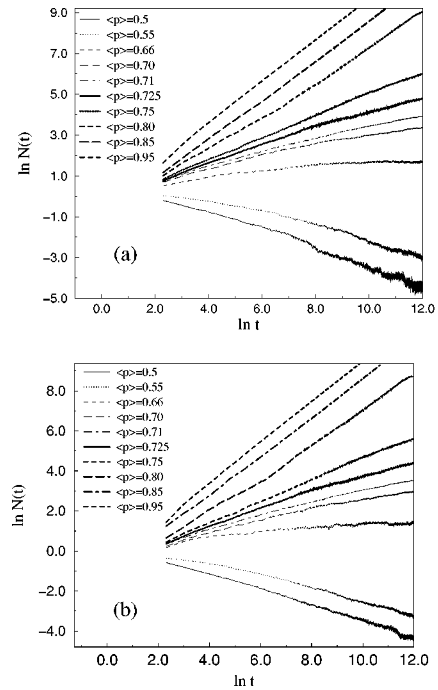

FIG. 7. Averaged total number of particles for spreading experiments, with different values of $\langle p\rangle$, as a function of time for the disordered model (upper plot) and for the non-Markovian model (lower plot).

tion of $\langle p\rangle$, and the point at which it reaches zero for the first time corresponds to the critical point. The second possibility is that there is a discontinuous jump at the critical point, i.e., the curves in Fig. 3 would not be continuous; this would imply a nonzero value of θ at the transition point. Even though from our numerics it is not possible to resolve the previous dilemma, we are tempted to conclude that the first possibility is the right one. This argument is based on the small values of $\theta(\langle p\rangle)$ in the vicinity of the critical point, and relies on the fact that the slopes are always observed to change smoothly with $\langle p\rangle$. Therefore, no discontinuity ''jump'' is expected to occur. In any case, from the numerics, θ can be expressed at the critical point as $\theta(\langle p\rangle_c)=0.02\pm0.05$, with $\langle p\rangle_c=0.71\pm0.01$ (see below).

In Fig. 4, we plot the asymptotic density n as a function of $\langle p\rangle$, together with a power-law fit. The best fit is obtained taking $\langle p\rangle_c=0.705$ for the critical effective parameter, and gives $\beta=0.29\pm0.01$ for both the disordered and non-Markovian models. In Fig. 5, we check the consistency of our assumption on $\langle p\rangle_c$, by representing in a log-log plot $n^{1/\beta}$ as a function of $\Delta=\langle p\rangle-\langle p\rangle_c$, with $\beta=0.29$. The extrapolation to zero of $n^{1/\beta}$ gives $\langle p\rangle_c=0.71\pm0.01$, consistent with our previous assumption. Observe that the value of β we find is very different from the one obtained by Noest for a different distribution of impurities, $\beta=1.75\pm0.1$ [11]. We interpret this discrepancy as a consequence of the absence of universality predicted by the field-theory analysis.

From the previous analysis (which agrees with the scaling laws we obtained theoretically) we can extract the following striking conclusion: as β assumes a finite value and θ is compatible with zero, using the scaling relation $\theta=\beta/\nu_t$ we obtain that either ν_t is infinity or takes an extremely large value. Observe that Noest measured $\nu_t=4.0\pm0.5$ [11], which is an atypically large value. In fact, an analogous result was obtained in the two-dimensional version of the model [16]; Dickman and Moreira showed that as a matter of fact the exponent ν_t is not even defined. This is a straightforward consequence of the fact that the correlation functions do not decay exponentially in the absorbing phase, but as a power law, i.e, there is no associated characteristic time, and therefore ν_t is undefined (or formally $\nu_t=\infty$).

In order to explore this issue further, we have measured the two-time correlation functions, $\langle n(t_0)n(t_0+t)\rangle-\langle n(t\to\infty)\rangle^2$ for large times and different values of $\langle p\rangle$ for the disordered model. The results are presented in Fig. 6. First we observe that in all the cases, i.e., beyond, below, and at the critical point, we obtain power-law behaviors, and therefore there is no characteristic time scale. Second, for a fixed value of $\langle p\rangle$ and varying t_0, we observe different transient regimes, but the asymptotic behavior does not depend on t_0 for large enough times. This indicates that the model does not exhibits aging [30]; therefore, even though the field theory representing the model is non-Markovian (i.e., the

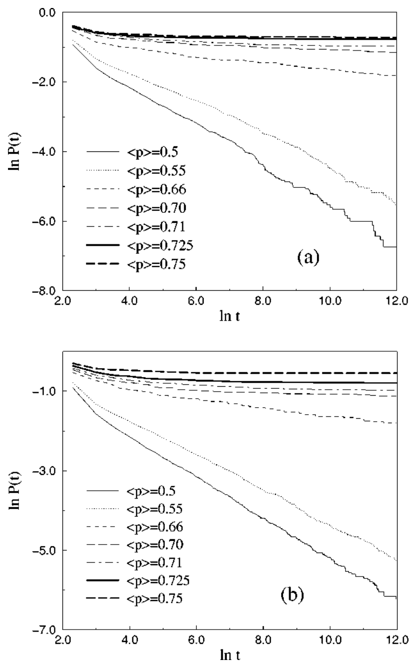

FIG. 8. Surviving probability for spreading experiments, with different values of $\langle p\rangle$, as a function of time for the disordered model (upper plot), and for the non-Markovian model (lower plot).

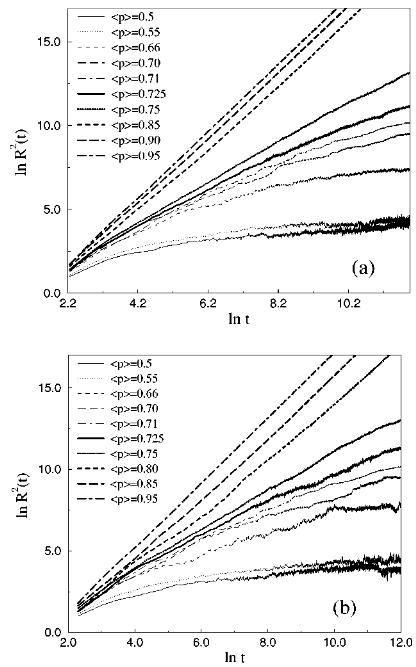

FIG. 9. Averaged square distance from the initial seed, with different values of $\langle p\rangle$, in a spreading experiment as a function of time for the disordered model (upper plot), and for the non-Markovian model (lower plot).

two-time correlation functions cannot be expressed only as a function of the time difference), the system relaxes to an aging-free state.

This analysis can be interpreted as further supporting the guess that $\theta=0$ at the critical point. Otherwise, using the scaling relations, we would obtain a finite ν_t and consequently an exponential decay of the two-time correlation function.

B. Spreading

In Fig. 7, 8, and 9, we present the evolution of the magnitudes defined in Eq. (3) for the spreading experiments. In Tables I and II, we give a summary of the values of all the scaling exponents for the magnitudes we have studied, including the exponent θ, related to homogeneous initial conditions, together with a checking of the scaling relations between the exponents.

Observe that the three magnitudes $N(t)$, $P_s(t)$, and $R^2(t)$ present generic power-law decays. In Tables I and II, we show the values of the associated exponents η, δ, and z for different values of $\langle p\rangle$. Note that all the scaling laws predicted in Sec. IV A are satisfied generically within the accuracy limits. In particular, it is important to note that right at the critical point and in the active phase we obtain a value of δ compatible with $\delta=0$, and therefore satisfying the predicted scaling relation, $\theta=\delta=0$. As a by-product, we obtain a confirmation of a result obtained some time ago in Ref. [21]. These authors demonstrated that an impure version of the one-dimensional contact process exhibits an intermediate phase, i.e., a region in the active phase in which R^2 grows slower that t^2. This is accordingly called the *sublinear regime* [31]. We observe sublinear growth in the entire active phase; only in the limit $\langle p\rangle=1$ $(a\to\infty)$ do we obtain linear growth, i.e., the intermediate phase coincides with the active phase (and also seems to extend to the absorbing phase). Therefore, the presence of such a sublinear regime seems to be a generic feature of impure one-dimensional systems with

TABLE I. Values of the scaling exponents for different values of p (disordered model).

p	η	δ	z	θ	$\eta+\delta-dz/2$	$\eta-dz/2$	$\delta-\theta$
0.5	-0.52 ± 0.02	0.61 ± 0.02	0.12 ± 0.07	0.57 ± 0.02	0.03 ± 0.07		0.04 ± 0.04
0.55	-0.32 ± 0.02	0.48 ± 0.02	0.14 ± 0.07	0.44 ± 0.02	0.09 ± 0.07		0.04 ± 0.04
0.66	-0.03 ± 0.01	0.10 ± 0.01	0.16 ± 0.06	0.13 ± 0.02	-0.01 ± 0.04		-0.03 ± 0.03
0.70	0.19 ± 0.02	0.05 ± 0.01	0.57 ± 0.02	0.02 ± 0.05	-0.04 ± 0.04		0.03 ± 0.06
0.71	0.25 ± 0.02	0.0 ± 0.01	0.58 ± 0.02	0.02 ± 0.05		-0.04 ± 0.03	-0.02 ± 0.06
0.725	0.35 ± 0.02	0	0.72 ± 0.02	0		-0.01 ± 0.03	0
0.75	0.53 ± 0.02	0	1.10 ± 0.01	0		-0.02 ± 0.03	0
0.8	0.92 ± 0.02	0	1.79 ± 0.01	0		0.02 ± 0.03	0
0.85	0.99 ± 0.02	0	1.99 ± 0.01	0		-0.01 ± 0.03	0
0.95	1.00 ± 0.02	0	2.00 ± 0.01	0		0.0 ± 0.03	0

TABLE II. Values of the scaling exponents for different values of p (non-Markovian model).

p	η	δ	z	θ	$\eta+\delta-dz/2$	$\eta-dz/2$	$\delta-\theta$
0.5	-0.50 ± 0.02	0.54 ± 0.02	0.10 ± 0.08	0.57 ± 0.02	-0.01 ± 0.08		-0.03 ± 0.04
0.55	-0.37 ± 0.02	0.43 ± 0.02	0.12 ± 0.08	0.43 ± 0.02	0.0 ± 0.08		0.0 ± 0.04
0.66	-0.08 ± 0.02	0.10 ± 0.01	0.14 ± 0.07	0.11 ± 0.02	-0.05 ± 0.06		-0.01 ± 0.03
0.70	0.19 ± 0.02	0.03 ± 0.01	0.57 ± 0.02	0.02 ± 0.04	-0.06 ± 0.04		0.01 ± 0.05
0.71	0.24 ± 0.02	0.0 ± 0.01	0.58 ± 0.02	0.01 ± 0.04		-0.05 ± 0.03	-0.01 ± 0.05
0.725	0.34 ± 0.02	0	0.81 ± 0.04	0		-0.06 ± 0.04	0
0.75	0.47 ± 0.02	0	1.03 ± 0.02	0		-0.04 ± 0.03	0
0.8	0.93 ± 0.02	0	1.79 ± 0.01	0		0.04 ± 0.02	0
0.85	0.99 ± 0.02	0	1.96 ± 0.01	0		0.01 ± 0.02	0
0.95	1.00 ± 0.02	0	2.00 ± 0.01	0		0.0 ± 0.02	0

absorbing states. Our results could be compared with those obtained by Noest for a different impurity distribution [12]. This author showed, at the critical point, that $\hat{d}=1.28\pm0.03$. Using Eq. (20), this implies $\eta+\delta=0.28\pm0.03$, to be compared with the value 0.25 ± 0.02 that we measure (Tables I and II). On the other hand, for exponent z, Noest measured $z=1.44\pm0.06$ (using the relation $z=2\nu_x/\nu_t$), and we obtain $z=0.58\pm0.02$, again indicating a high degree of nonuniversality (Tables I and II).

As a last observation, we want to point out that the curves for $\langle R^2(t)\rangle$ in the absorbing phase (see Fig. 9) do not seem to have reached their stationary value in the time scale under consideration. Thus the values of z we give are just a rough estimation, since error bars are quite large. Our results could be asymptotically compatible with $z=0$. Observe, also, that the combination $\eta+\delta$ in the absorbing phase gives a small exponent that could also be compatible with zero asymptotically. In any case, all the predicted scaling relations among exponents are perfectly satisfied both above and below the critical point.

VII. CONCLUSIONS

We have studied the disordered contact process under different perspectives. First, we have mapped it into a pure model with memory, that reproduces all the phenomenology of the original model. From this non-Markovian model, we write down a simple field theory, that is in the same universality class as one presented previously for the model with Gaussian-distributed quenched disorder. Using this field theory we have derived a set of scaling relations not only at the critical point but also in the active and absorbing phases where scale invariance is also observed. Our theoretical predictions are confirmed in extensive Monte Carlo simulations. In particular, we have shown the equivalence of the disordered and non-Markovian models, the generic presence of scale invariance, and the existence of a sublinear-growth regime. We also emphasized the absence of a characteristic time scale, and verified all the predicted scaling laws.

In a future work we plan to investigate the non-Markovian model further in order to obtain some complementary analytical results. In particular, we aim to apply real-space renormalization methods to the one-dimensional model, and plan to propose an investigation of the generic scale invariance from a renormalization point of view.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge useful discussions with Ron Dickman. This work was partially supported by the European Union through a grant to M.A.M. (Contract No. ERBFMBICT960925).

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